Proof by Induction

A method of proving something is true for all Integers values.

If P(1) is true

AND P(k) implies P(k + 1) is true

THEN P(n) is true for all Ints greater than or equal to 1

Eg

If person k gets a burger then person k + 1 also gets a burger

PROVE: 1 + 2 + 3…..+ n = n(n+1)/2 for all n elements of the natural numbers 🡺 n elements {1,2,3….etc}

1. PROVE TRUE FOR n = 1

1(1+1)/2 = 1(2)/2 = 1

STATEMENT IS TRUE

1. ASSUME TRUE FOR n = k

Assume true: 1 +2+ 3…….+k = k(k+1 )/2

1. PROVE TRUE FOR n = k + 1

1+2+3….k+1 = (k+1)(k+1+1)/2

🡺1+2+3…..+k +(k+1)

= k(k + 1) / 2 + (k + 1)

= k(k+1) + 2(k + 1) / 2

= k^2 + k + 2k + 2/2

(k + 1)(k + 1 + 1)/2

(k + 1)(k + 2)/2

K^2 + 3k + 2/2

Example :

Prove 3 + 6 + 9….+3n = 3n(n+1 )/2

1. Prove for n = 1

3( 1) (1+ 1) /2

3(2)/2

3

1. Assume for n = k

3(k)( k+1) /2

3(k^2 + k)/2

1. Prove for n = k + 1

3(k+1)(k+1+1)/2

3(k+ 1) ( k+ 2) /2

3( k^2 + 3k + 2)/2

3(k^2 + k + 2k + 2)/2

3(k^2 + k) + 3(2k + 2)/2

3(k^2 + k)/2 + 3(k+1)

Example:

Prove 11^n – 6 is divisible by five

Let n = 1

11^1 – 6 / 5

* 11 – 6 = 5/5 = 1

Let n = k and we wil assume the equation is true

(11^k – 6 )/ 5 = M

11^k = 5M + 6

Where M is an element if the set of elements

Prove N = k + 1

(11^(k+1) – 6)/5

((11^k \* 11) ) – 6)/5

((5M + 6) \* 11) – 6)/5

55M + 60/5

11M + 12

2(5M + 6) + 1

2(11^k) + 1

2(((11^k)-6)/5) + 1

Prove by induction

1^2 + 2^2 + 3^2 +………….+n^2 = n(n+1)(n+2)/6

N = 1

1(1+1)(1+2)/6

2\*3/6 = 1

Assume n = k

K(k+1)(k+2)/6

K(k^2 + 3k + 2)/6

Prove n = K + 1

(k+1) ((k+1)+1) ((k+1)+2) / 6

(k+1)(k+2)(k+3) / 6

(k^2 + 3k + 2)(k+3) / 6

K^3 + 3(k^2) + 2k + 6 + 3(k^2) + 9k + 6

K^3 + 6(k^2) + 11k + 12 / 6

K(k^2) + k(3k) + k(2) + 3(k^2) + 3(3k) + 3(2) + 3(2) / 6

K(k^2 + 3k + 2) + 3(k^2 + 3k + 2) + 6 /6

If n = k + 1

Then n – 1 = k

K(k^2 + 3k + 2)/6 + k+1